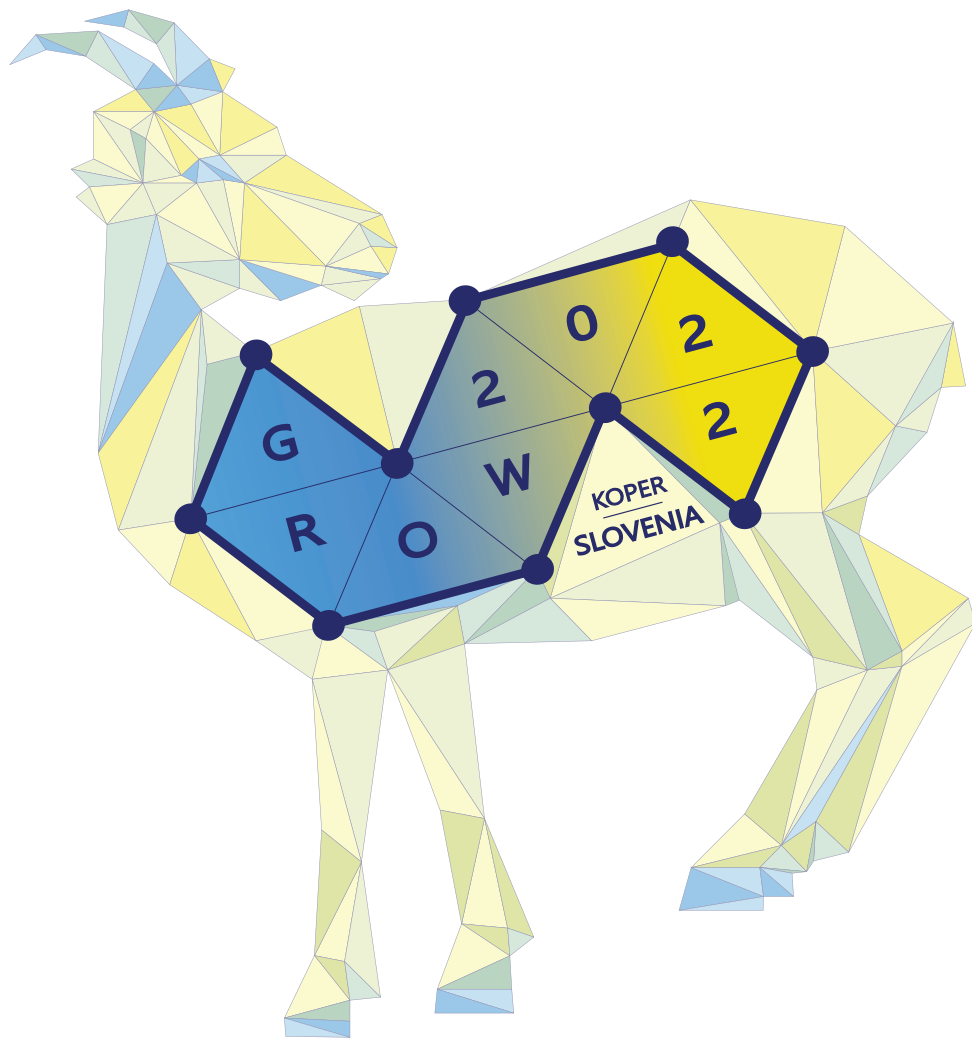


10th Workshop on Graph Classes, Optimization, and Width Parameters

GROW 2022

Book of Abstracts



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10th Workshop on Graph Classes, Optimization, and Width Parameters
GROW 2022: Book of Abstracts

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Preface

The *Workshop on Graph Classes, Optimization, and Width Parameters* (GROW) series of workshops brings together experts in both theoretical and practical issues to design new strategies for dealing with intractable graph problems. Its 10th edition, GROW 2022, was hosted by the University of Primorska, Faculty of Mathematics, Natural Sciences and Information Technologies, Koper, Slovenia, from September 19–22, 2022. The event was originally planned for September 2021, but it was decided to postpone it for a year because of the COVID-19 pandemic. This decision allowed to reduce the risks of an outbreak that would reduce dramatically the participation and, eventually, lead to a fully in-person event, as in all previous editions.

In this edition we enjoyed three distinguished keynote speakers: Marthe Bonamy, Bart M. P. Jansen and Bojan Mohar. We also had the pleasure to have a session to celebrate the work of Vadim Lozin because of his 60th birthday.

This booklet contains the short abstracts of the keynote and contributed talks given at GROW 2022. Together with the *Book of Open Problems*, they provide a glimpse of the breadth and excellent scientific quality of the research presented during the event. Nevertheless, they do not show the enjoyment of taking part of GROW, a unique event to meet colleagues and discuss research in an enjoyable, fruitful atmosphere.

Sergio Cabello and Martin Milanič
Chairs of GROW 2022

Tradition of the Workshop on Graph Classes, Optimization, and Width Parameters:
GROW 2022 – Koper (Slovenia)
GROW 2019 – Wien (Austria)
GROW 2017 – Toronto (Canada)
GROW 2015 – Aussois (France)
GROW 2013 – Santorini Island (Greece)
GROW 2011 – Daejeon (South Korea)
GROW 2009 – Bergen (Norway)
GROW 2007 – Eugene (USA)
GROW 2005 – Prague (Czech Republic)
GROW 2001 – Barcelona (Spain)

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Keynote talks

One graph to rule them all: forbidden structures and universal graphs

Marthe Bonamy

CNRS, University of Bordeaux

Consider all planar graphs on n vertices. What is the smallest graph that contains them all as induced subgraphs? In this talk, we will gently introduce the audience to the notion of so-called universal graphs (graphs containing all graphs of a given family as induced subgraphs), and focus on the case of graph classes defined by forbidden structures. We present positive and negative results both in dense graphs and in sparse graphs. The audience will also have the answer to the question at the beginning of this abstract, recently established in a breakthrough paper of Dujmović, Esperet, Joret, Gavaille, Micek and Morin.

Search-Space Reduction Beyond Kernelization

Bart M. P. Jansen

Eindhoven University of Technology

The framework of kernelization gives a mathematical model for the rigorous analysis of preprocessing, aimed at showing that any instance with parameter k can efficiently be reduced to an equivalent one whose size depends only on k . Kernelization serves as a useful tool to obtain FPT algorithms, since any brute-force algorithm solves the reduced instance in time depending only on k . However, from the definition of kernelization it is not clear why kernelization would lead to significant speed-ups when the reduced instance is not solved by brute force, but by a fixed-parameter tractable algorithm whose running time is governed by the value of the parameter. To speed up such algorithms, it is the parameter controlling the size of the search space which should decrease, rather than the encoding size of the input. The discrepancy between reducing the instance size versus decreasing the search space is the focus of this talk. The quest for preprocessing algorithms that reduce the search space leads to a new type of algorithmic and graph-theoretical questions. The talk gives an introduction to this budding line of inquiry by combining examples from recent work with open problems for future research.

On the structure of crossing-critical graphs

Bojan Mohar

Simon Fraser University and IMFM

Graphs that are critical for the crossing number (their crossing number is at least k but removing any edge decreases the crossing number below k) have bounded path-width. This was proved by Hlineny in 2003. So what else can be said about the structure of crossing-critical graphs for a fixed k ? It turns out that there is much more, and this will be the main focus of the presentation.

This is joint work with Zdenek Dvorak and Petr Hlineny.

Contributed talks

Bounds for the twin-width of graphs

Jungho Ahn

KAIST / IBS DIMAG

Bonnet, Kim, Thomassé, and Watrigant [J. ACM 2022] introduced the *twin-width* of a graph. We show that the twin-width of an n -vertex graph is less than $(n + \sqrt{n} \ln n + \sqrt{n} + 2 \ln n)/2$, and the twin-width of an m -edge graph for a positive m is less than $\sqrt{3m} + m^{1/4} \sqrt{\ln m} / (4 \cdot 3^{1/4}) + 3m^{1/4}/2$. Conference graphs of order n (when such graphs exist) have twin-width at least $(n-1)/2$, and we show that Paley graphs achieve this lower bound. We also show that the twin-width of the Erdős-Rényi random graph $G(n, p)$ with $1/n \leq p \leq 1/2$ is larger than $2p(1-p)n - (2\sqrt{2} + \varepsilon) \sqrt{p(1-p)n \ln n}$ asymptotically almost surely for any positive ε . Lastly, we calculate the twin-width of random graphs $G(n, p)$ with $p \leq c/n$ for a constant $c < 1$, determining the thresholds at which the twin-width jumps from 0 to 1 and from 1 to 2.

Joint work with Kevin Hendrey in IBS, Donggyu Kim in KAIST and IBS, and Sang-il Oum in IBS and KAIST. Preprint available at <https://arxiv.org/abs/2110.03957>.

Characterizing Graph Classes with Convexity

Jesse Beisegel

Brandenburg University of Technology

Some well known graph classes, such as chordal graphs, can be described by an underlying abstract convexity. In fact, it can be shown that a graph is chordal if and only if its monophonic convexity is a convex geometry, i.e., if every convex set is the convex hull of its extreme points. We present some known results on this topic in a common form, so as to give a homogeneous representation of a very disparate field. Using these standardized definitions, we study other convexities and use these to characterize some well known classes of graphs via a convex geometry. Furthermore, we present some new results on the Carathéodory number of interval graphs and also give an account of everything that is known in this context on AT-free graphs, including new results the structure of the intervals of this class.

Reconfiguration of Vertex Colouring and Forbidden Induced Subgraphs

Manoj Belavadi

Department of Mathematics, Wilfrid Laurier University, Waterloo, Canada

The reconfiguration graph of the k -colourings, denoted $\mathcal{R}_k(G)$, is the graph whose vertices are the k -colourings of G and two colourings are adjacent in $\mathcal{R}_k(G)$ if they differ in colour on exactly one vertex. We investigate the connectivity and diameter of $\mathcal{R}_{k+1}(G)$ for a k -colourable graph G restricted by forbidden induced subgraphs. We show that $\mathcal{R}_{k+1}(G)$ is connected for every k -colourable H -free graph G if and only if H is an induced subgraph of P_4 or $P_3 + P_1$.

We also start an investigation into this problem for (H_1, H_2) -free graphs, where H_1 and H_2 are 4-vertex graphs other than P_4 and $P_3 + P_1$. We show that if G is a k -colourable $(2K_2, C_4)$ -free graph, then $\mathcal{R}_{k+1}(G)$ is connected and has diameter at most $4n$. Furthermore, we show that if $\mathcal{R}_{k+1}(G)$ is connected for every k -colourable (H_1, H_2) -free graph G , then either H_1 or H_2 is isomorphic to $2K_2$.

This is joint work with Kathie Cameron and Owen Merkel.

A preprint is available at <https://arxiv.org/abs/2206.09268>.

Hardness constructions for graph covering problems with semi-edges

Jan Bok

Faculty of Mathematics and Physics, Charles University, Prague

I will talk about our recent line of research focused on determining the complexity of the problem of finding a covering projection between graphs. A graph covering projection, also known as a locally bijective homomorphism, is a mapping between vertices and edges of two graphs which preserves incidencies and is a local bijection. This notion stems from topological graph theory, but has also found applications in combinatorics and theoretical computer science.

I will mainly focus on various hardness constructions we utilized and introduced in our papers from MFCS 2021 (dichotomy for the H -COVER problem with one-vertex and two-vertex targets) and IWOCA 2022 (list version of the H -COVER problem). In particular, the talk will highlight the importance of bipartite graphs in these constructions.

Joint work with Jiří Fiala, Petr Hliněný, Nikola Jedličková, Jan Kratochvíl, and Paweł Rzażewski.

Locally checkable problems parameterized by treewidth, clique-width and mim-width

Carolina Lucía Gonzalez
CONICET - Universidad de Buenos Aires

Locally checkable problems are partitioning (or, equivalently, coloring) problems where the solution can be verified by checking some local property for each vertex, that is, a property involving only the vertex and its neighborhood. This is the case of stable set, dominating set and k -coloring, among others. We can consider that each of these problems has an associated set of colors and a *check function*: a function that takes as input a vertex v of the graph and a coloring of the neighborhood of v , and outputs TRUE or FALSE. In this context, a *proper coloring* of a graph is defined as a coloring c of the vertices such that, for every vertex v , the check function applied to v and the restriction of c to the neighborhood of v outputs TRUE. We can also consider a set of weights with a total order, and associate a weight to each pair of vertex and color. The weight of a coloring c is then naturally obtained by combining the weights of the pairs $(v, c(v))$. Finally, we want to find the minimum weight of a proper coloring of the input graph.

Since many locally checkable problems are hard on general graphs, it is of interest to determine under which conditions we can efficiently solve them for a given class of graphs. In this work, we study the different restrictions we need to impose to the check function and the number of colors in order to obtain FPT or XP algorithms when parameterized by different width measures (in particular, treewidth, clique-width and mim-width).

This is joint work with Flavia Bonomo-Braberman [2], Narmina Baghirova, Bernard Ries and David Schindl [1], and Felix Mann [3].

References

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- [2] F. Bonomo-Braberman and C.L. Gonzalez. A new approach on locally checkable problems. *Discrete Applied Mathematics*, 314:53–80, 2022.
- [3] C.L. Gonzalez and F. Mann. On d -stable locally checkable problems on bounded mim-width graphs. arXiv:2203.15724 [cs.DM], 2022.

LexCycle on Modules

Michel Habib

IRIF

The LexCycle Conjecture states that $\text{LexCycle}(G) = 2$ for all G cocomparability. It has been proven for various sub-classes of cocomparability graphs (interval, domino-free, cographs...). In this work, we prove the LexCycle conjecture for a wide class of cocomparability graphs, namely those closed under modular decomposition. This means it is sufficient to prove the conjecture for prime graphs only. What we prove in particular is the class of cocomparability graphs with $\text{LexCycle} = 2$ is closed under the addition of a new (false or true) twin vertex. Therefore to conclude the proof of the Conjecture, it remains now to prove it for prime graphs. To this end, we managed to prove it for P_4 -sparse and P_4 -reducible cocomparability graphs. The results of this work are also of independent interest since modular decomposition algorithms are often based on LexBFS as a preprocessing step.

Joint work with Lalla Mouatadid.

Generating Strongly 2-Connected Digraphs

Meike Hatzel

NII, Tokyo

Tutte proved that all 3-connected graphs can be generated from the set of wheels by two simple graph operations. In 2001 McCuaig presented a way to generate all braces, that is all connected bipartite subgraphs that allow for a perfect matching, have at least six vertices and any two non-adjacent edges are contained in a perfect matching. This can be done using only a small set of simple graph operations. Braces are a fundamental tool in matching theory and can be applied in solving Polyas Permanent Problem. In this talk we consider an analogue of the above results in the setting of directed graphs and butterfly minors. We present a way to generate all strongly 2-connected digraphs from a base set of digraphs using a fixed set of operations. The construction also allows us to formulate a splitter theorem, in the flavour of Seymour's splitter theorem, for strongly 2-connected digraphs.

This is joint work with Stephan Kreutzer, Evangelos Protopapas, Florian Reich, Giannos Stamoulis and Sebastian Wiederrecht.

Critical $(P_3 + \ell P_1)$ -free Critical $(P_3 + \ell P_1)$ -free and critical (gem, co-gem)-free graphs

Chinh T. Hoang

Wilfrid Laurier University

A graph G is k -vertex-critical if $\chi(G) = k$ but $\chi(G - v) < k$ for all $v \in V(G)$ where $\chi(G)$ denotes the chromatic number of G . We show that there are only finitely many k -critical $(P_3 + \ell P_1)$ -free graphs for all k and all ℓ . Together with previous results, the only graphs H for which it is unknown if there are an infinite number of k -vertex-critical H -free graphs is $H = (P_4 + \ell P_1)$ for all $\ell \geq 1$. We consider a restriction on the smallest open case, and show that there are only finitely many k -vertex-critical (gem, co-gem)-free graphs for all k , where $\text{gem} = \overline{P_4 + P_1}$. To do this, we show the stronger result that every vertex-critical (gem, co-gem)-free graph is either complete or a clique expansion of C_5 . This characterization allows us to give the complete list of all k -vertex-critical (gem, co-gem)-free graphs for all $k \leq 11$. This is joint work with Tala Abuadas, Ben Cameron and Joe Sawada.

List homomorphism problem for signed graphs

Nikola Jedličková

Faculty of Mathematics and Physics, Charles University, Prague, Czech Republic

In the talk, we are interested in classifying the computational complexity of the list homomorphism problem for signed graphs. For a fixed signed graph \widehat{H} , the list homomorphism problem asks whether an input signed graph \widehat{G} with lists $L(v) \subseteq V(\widehat{H}), v \in V(\widehat{G})$, admits a homomorphism f to \widehat{H} with all $f(v) \in L(v), v \in V(\widehat{G})$. Although a dichotomy classification for homomorphisms of signed graphs has been already established, the list version remains open.

We shall provide both polynomial and NP-complete cases of this problem, obtained as partial results in our effort towards solving the full dichotomy of the problem. Firstly, I will briefly sketch our results when \widehat{H} is a signed tree and when \widehat{H} is a special case of weakly balanced irreflexive signed graphs. Then as a generalisation of this results I will show a full dichotomy for weakly balanced irreflexive signed graphs (previously conjectured by Kim and Siggers).

Polynomial algorithm to compute the toughness of graphs with bounded treewidth

Gyula Y. Katona

Budapest University of Technology and Economics

Let t be a real number. A graph is called t -tough if the removal of any vertex set S that disconnects the graph leaves at most $|S|/t$ components. The toughness of a graph is the largest t for which the graph is t -tough.

Computing the toughness of a given graph is most likely difficult. Unless $P=NP$, there is no polynomial algorithm for it. It remains difficult for regular graphs and bipartite graphs, even for regular bipartite and k -connected bipartite graphs.

On the other hand, there are polynomial algorithms to compute the toughness for split graphs, interval graphs, claw-free and $2K_2$ -free graphs.

Our new result is a polynomial algorithm to compute the toughness of graphs with bounded treewidth.

The complexity of computing optimum labelings for temporal connectivity

Nina Klobas

Durham University

A graph is temporally connected if there exists a strict temporal path, i.e. a path whose edges have *strictly increasing* labels, from every vertex u to every other vertex v . In this talk we study *temporal design* problems for undirected temporally connected graphs. The basic setting of these optimization problems is as follows: given a connected undirected graph G , what is the smallest number $|\lambda|$ of time-labels that we need to add to the edges of G such that the resulting temporal graph (G, λ) is temporally connected? As it turns out, this basic problem, called Minimum Labeling (ML), can be optimally solved in polynomial time. However, exploiting the temporal dimension, the problem becomes more interesting and meaningful in its following variations. First we consider the problem Minimum Aged Labeling (MAL) of temporally connecting the graph when we are given an upper-bound on the allowed age (i.e. maximum label) of the obtained temporal graph (G, λ) . Second we consider the problem Minimum Steiner Labeling (MSL), where the aim is now to have a temporal path between any pair of “important” vertices which lie in a subset $R \subseteq V$, which we call the terminals. This relaxed problem resembles the Steiner Tree problem in static (i.e. non-temporal) graphs. However, due to the requirement of strictly increasing labels in a temporal path, Steiner Tree is not a special case of MSL. Finally we consider the age-restricted version of MSL, namely Minimum Aged Steiner Labeling (MASL). Our main results are threefold: we prove that (i) MAL becomes NP-complete on undirected graphs, while (ii) MASL becomes $W[1]$ -hard with respect to the number $|R|$ of terminals. On the other hand we prove that (iii) although the age-unrestricted problem MSL remains NP-hard, it is in FPT with respect to the number $|R|$ of terminals. That is, adding the age restriction, makes the above problems strictly harder (unless $P=NP$ or $W[1]=FPT$).

Computing Tree Decompositions with Small Independence Number

Tuukka Korhonen
University of Bergen

The independence number of a tree decomposition is the maximum of the independence numbers of the subgraphs induced by its bags. The tree-independence number of a graph is the minimum independence number of a tree decomposition of it. Several NP-hard graph problems, like maximum weight independent set, can be solved in time $n^{\mathcal{O}(k)}$ if the input graph is given with a tree decomposition with independence number at most k . However, it was an open problem if tree-independence number could be computed or approximated in $n^{f(k)}$ time, for some function f , and in particular it was not known if the requirement to have the decomposition as an input could be removed from the algorithms using tree-independence number.

We resolve the main open problems about the computation of tree-independence number. First, we give an algorithm that given an n -vertex graph G and an integer k , in time $2^{\mathcal{O}(k^2)}n^{\mathcal{O}(k)}$ either outputs a tree decomposition of G with independence number at most $8k$, or determines that the tree-independence number of G is larger than k . This implies $2^{\mathcal{O}(k^2)}n^{\mathcal{O}(k)}$ time algorithms for various problems parameterized by tree-independence number k without needing the decomposition as an input. Then, we show that the exact computing of tree-independence number is para-NP-hard, in particular, that for every constant $k \geq 4$ it is NP-hard to decide if a given graph has tree-independence number at most k .

Joint work with Clément Dallard, Fedor V. Fomin, Petr A. Golovach, and Martin Milanič.

Computational complexity of the list version of graphs covers

Jan Kratochvil

Charles University, Prague

A graph covering projection from a graph G to a graph H is a pair of incidence preserving mappings between the vertices and between the edges of G and H which is locally bijective on the neighborhoods of all vertices. This notion stems from topological graph theory, but has found applications in models of computation on one hand, and constructions of highly symmetric graphs on the other one. With computer science applications in mind, we are interested in the computational complexity of the problem, referred to as H -COVER, "Given an input graph G , does it cover H ?". The quest for a complete characterization of its complexity parameterized by the target graph H was started by Bodlaender [1988] and Abello, Fellows, Stilwell [1991]. Despite a considerable effort, the final answer is not at sight yet. One of the most general results says that H -COVER is NP-complete whenever H is a connected simple regular graph of valency at least 3 (Kratochvil, Proskurowski, Telle [1997] and Fiala [2000]).

Since the notion of graph covers stems from topological graph theory, it is natural to consider (multi)graphs allowing multiple edges and loops (these were considered already by Abello et al.), colored mixed graphs (considered by Kratochvil, Proskurowski, Telle [1997]), and very recently also allowing semi-edges (Bok et al. [2021]). We revisit the proof of Kratochvil, Proskurowski, Telle [1997] and show how it can be adjusted in certain situations for multigraphs (including semi-edges). One of the results obtained in this way is the following.

Theorem 1. *If H is a connected regular (multi)graph with at least one semi-simple vertex, then LIST- H -COVER is NP-complete even for simple input graphs.*

We stress the role of simple input graphs in the theorem. In all cases when the complexity of H -COVER is known, it turns out that the problem is either polynomial time solvable for arbitrary (multi)graphs on input, or NP-complete even for simple input graphs. The hope that this will be the case for all graphs H is referred to as the **Strong Dichotomy Conjecture**. Theorem 1 accompanied by several ad hoc reductions implies the following.

Theorem 2. *Cubic multigraphs obey the Strong Dichotomy.*

The talk is based on joint work with Jan Bok, Jiri Fiala, Nikola Jedlickova, and Pawel Rzazewski and was accepted and presented at IWOCA 2022 in Trier.

Constant-time connectivity tests

Philipp K. Krause

Property testing is concerned with extremely fast (constant-time or other sublinear) algorithms for approximate decision-making. While the runtime of constant-time algorithms does not depend on the size of the input, the runtime depending on parameters, such as the average degree of the input graph, or on the maximum allowed error of the output are still the subject of research.

We implemented constant-time graph algorithms for testing connectivity, 2-edge connectivity and 3-edge connectivity for sparse graphs. We implemented an estimate of the number of connected components, estimates of the distance to connectivity, 2-edge-connectivity and eulerianity, tolerant testers for connectivity, 2-edge-connectivity and eulerianity. We also present an algorithm for estimating the distance to 3-edge-connectivity and a tolerant tester for 3-edge-connectivity that have not been implemented yet. For connectivity and eulerianity our approach has better worst-case runtime than previous approaches (and the same expected runtime). For 2-edge-connectivity and 3-edge-connectivity, our approaches have better runtime than the previously known algorithm for general k -edge-connectivity. These advantages carry over to the tolerant testers.

Full, compileable C source of our implementation can be found as free software at <http://zshg.sourceforge.net>.

On Structural Parameterizations of Continuous Facility Location Problems on Graphs

Stefan Lendl
University of Graz

We study continuous facility location problems on a graph where all edges have unit length and where the facilities may also be positioned in the interior of the edges. In the covering variant, the goal is to cover the entire graph with a minimum number of facilities with covering range $\delta > 0$. In other words, we want to position as few facilities as possible subject to the condition that every point on every edge is at distance at most δ from one of these facilities. On the contrary, in the dispersion variant, the goal is to position as many facilities as possible subject to the condition that every two facilities have distance at least δ from each other. In recent work, we analyzed the computational complexity of both problems for all constant $\delta > 0$ and also their parameterized complexity for the natural parameter solution size.

We continue this line of investigation and consider the parameterized complexity with respect to the structure of the input graph. A major technical contribution is a new efficient procedure to ‘round-up’ distance δ . It transforms a δ -dispersed set S into a δ^* -dispersed set S^* of same size where distance δ^* is a slightly larger rational $\frac{a}{b}$ with a numerator a upper bounded by the longest (not-induced) path in the input graph. A similar rounding result can also be shown for the covering variant. Based on this rounding procedure we derive a number of algorithmic results for graph parameters like treewidth, treedepth and obtain matching lower bounds on its time complexity under ETH.

In the direction of dense graphs, we consider the neighborhood diversity of the input graph. We present an FPT algorithm for the neighborhood diversity. To obtain this result, we observe that it suffices to consider well-structured solutions only. Based on this canonical form, we identify two optimization problems, a linear program and a b -matching, whose combination yields a maximum dispersed set.

Based on joint work with Tim A. Hartmann. Some of the results are available in the preprint at <https://arxiv.org/abs/2206.11337>.

Graph parameters, implicit representations and factorial properties

Vadim Lozin
University of Warwick

A representation of an n -vertex graph G is implicit if it assigns to each vertex of G a binary code of length $\mathcal{O}(\log n)$ so that the adjacency of two vertices is a function of their codes. A necessary condition for a hereditary class X of graphs to admit an implicit representation is that X has at most factorial speed of growth. This condition, however, is not sufficient, as was recently shown in [H. Hatami, P. Hatami, The implicit graph conjecture is false, arXiv preprint arXiv:2111.13198]. Several sufficient conditions for the existence of implicit representations deal with boundedness of some parameters, such as degeneracy or clique-width. In this talk, we analyze more graph parameters and prove a number of new results related to implicit representation and factorial properties.

This is a joint work with Bogdan Alecu, Vladimir Alekseev, Aistis Atminas, Viktor Zamaraev.

Structural and algorithmic results for mim-width and sim-width

Andrea Munaro
University of Parma

Mim-width and sim-width are among the most powerful width parameters, with sim-width more powerful than mim-width, which is in turn more powerful than clique-width. A large number of NP-hard graph problems become polynomial-time solvable on graph classes whose mim-width is bounded and quickly computable. Hence, when solving such problems on special graph classes, it is helpful to know whether the graph class under consideration has bounded mim-width. We present summary theorems of the current state of the art for boundedness of mim-width for (H_1, H_2) -free graphs.

Contrary to mim-width, no algorithmic applications of boundedness of sim-width are known. In [Kang et al., A width parameter useful for chordal and co-comparability graphs, *Theoretical Computer Science*, 704:1-17, 2017], it is asked whether INDEPENDENT SET and 3-COLOURING are NP-complete on graphs of sim-width at most 1. We observe that, for each $k \in \mathbb{N}$, LIST k -COLOURING is polynomial-time solvable for graph classes whose sim-width is bounded and quickly computable. Moreover, we show that if the same holds for INDEPENDENT SET, then INDEPENDENT \mathcal{H} -PACKING is polynomial-time solvable for graph classes whose sim-width is bounded and quickly computable. This problem is a common generalisation of INDEPENDENT SET, INDUCED MATCHING, DISSOCIATION SET and k -SEPARATOR.

We finally investigate relations between mim-width and sim-width and other width parameters on restricted graph classes. In particular, we show the following results. For any graph class \mathcal{G} , sim-width, mim-width and clique-width are equivalent on the class $L(\mathcal{G})$ of line graphs of graphs in \mathcal{G} , and are bounded therein if and only if \mathcal{G} has bounded treewidth. Sim-width, mim-width, clique-width, treewidth and tree-independence number are equivalent on the class of $K_{t,t}$ -subgraph-free graphs. Finally, we show that a graph with large tree-independence number either contains a large biclique as an induced subgraph or has large mim-width.

Based on joint works with N. Brettell, D. Paulusma and S. Yang.

The mixed search game against an agile and visible fugitive is monotone

Christophe Paul

CNRS, Université de Montpellier

We consider the mixed search game against an agile and visible fugitive. This is the variant of the classic fugitive search game on graphs where searchers may be placed to (or removed from) the vertices or slide along edges. Moreover, the fugitive resides on the edges of the graph and can move at any time along unguarded paths. The *mixed search number against an agile and visible fugitive* of a graph G , denoted $\text{avms}(G)$, is the minimum number of searchers required to capture to fugitive in this graph searching variant. Our main result is that this graph searching variant is *monotone* in the sense that the number of searchers required for a successful search strategy does not increase if we restrict the search strategies to those that do not permit the fugitive to visit an already “clean” edge. This means that mixed search strategies against an agile and visible fugitive can be polynomially certified, and therefore that the problem of deciding, given a graph G and an integer k , whether $\text{avms}(G) \leq k$ is in NP.

Our proof is based on the introduction of the notion of *tight bramble*, that serves as an obstruction for the corresponding search parameter. Our results imply that for a graph G , $\text{avms}(G)$ is equal to the Cartesian tree product number of G that is the minimum k for which G is a minor of the Cartesian product of a tree and a clique on k vertices.

Tree-layout based graph classes: the case of proper chordal graphs.

Evangelos Protopapas

LIRMM, Université Montpellier, CNRS

Many standard graph classes are known to be characterized by means of layouts (a permutation of its vertices) excluding some patterns. Important such graph classes are proper interval graphs, interval graphs, chordal graphs but also permutation graphs, (co-)comparability graphs and so on.

For example, a graph $G = (V, E)$ is an interval graph if and only if G has a layout \mathbf{L} such that for every triple of vertices such that $x \prec_{\mathbf{L}} y \prec_{\mathbf{L}} z$, if $xz \in E$, then $xy \in E$. We call such a layout an *interval layout*. Proper interval graphs are characterized by excluding *indifference triples*, defined as triples $x \prec_{\mathbf{L}} y \prec_{\mathbf{L}} z$ such that if $xz \in E$, then $xy \in E$ and $yz \in E$.

In this talk, we investigate the concept of *tree-layouts*. A tree-layout \mathbf{T} of a graph $G = (V, E)$ is a rooted tree (T, r) equipped with a one-to-one mapping between V and the node of T such that for every edge $xy \in E$, either x is an ancestor of y , denoted $x \prec_{\mathbf{T}} y$, or y is an ancestor of x . Excluding a pattern in a tree-layout is defined similarly as excluding a pattern in a layout, but now using the ancestor relation. It can be easily observed that chordal graphs are characterized by the existence of a tree-layout that excludes the interval pattern discussed above. As a proof of concept, we show that excluding indifference triples in tree-layouts yields a natural graph class of *proper chordal graphs*. We will position proper chordal graphs with respect to other known graph classes and explore its structural and algorithmic aspects.

Joint work with Christophe Paul (LIRMM, CNRS, Université Montpellier).

Evaluating Restricted First-Order Counting Properties on Nowhere Dense Classes and Beyond

Peter Rossmanith
RWTH Aachen University

It is known that first-order logic with some counting extensions can be efficiently evaluated on graph classes with bounded expansion, where depth- r minors have constant density. To be more precise, the formulas are of the form $\exists x_1 \dots x_k \#y \phi(x_1, \dots, x_k, y) > N$, where ϕ is an FO-formula. If ϕ is quantifier-free, we can extend this result to *nowhere dense* graph classes with an almost linear FPT run time. Lifting this result further to slightly more general graph classes, namely almost nowhere dense classes, where the size of depth- r clique minors is subpolynomial, is impossible unless $\text{FPT} = \text{W}[1]$. On the other hand, in almost nowhere dense classes we can approximate such counting formulas with a small additive error.

In particular, it follows that partial covering problems, such as partial dominating set, have fixed parameter algorithms on nowhere dense graph classes with almost linear running time.

Joint work with Jan Dreier and Daniel Mock.

Semi-Proper Interval Graphs

Robert Scheffler
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Chordal graphs, i.e., graphs whose induced cycles are triangles, can be characterized via perfect elimination orderings (PEOs). In these vertex orderings every vertex is simplicial in the graph that is induced by the vertices that are to the left of it in the ordering. We strengthen this definition by forcing consecutive vertices in the ordering to be adjacent. The resulting vertex orderings characterize a graph class that lies between the classes of proper interval graphs and interval graphs. Thus, we name these graphs semi-proper interval graphs. We present alternative characterizations using special interval models and clique orderings. Furthermore, we study the problem of recognizing semi-proper interval graphs. We will see that the usage of PQ-trees leads to a linear-time recognition algorithm, while known multi-sweep graph search approaches used for (proper) interval graphs do not work in linear time. This latter result also leads to some insights on multi-sweep graph searches for proper interval graph recognition. Finally, we show that strong results on the existence of Hamiltonian paths and cycles in proper interval graphs can be generalized to semi-proper interval graphs.

Model-Checking for First-Order Logic with Disjoint Paths Predicates in Proper Minor-Closed Graph Classes

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The *disjoint paths logic*, FOL+DP, is an extension of First Order Logic (FOL) with the extra atomic predicate $\text{dp}_k(x_1, y_1, \dots, x_k, y_k)$, expressing the existence of internally vertex-disjoint paths between x_i and y_i , for $i \in \{1, \dots, k\}$. This logic can express a wide variety of problems that escape the expressibility potential of FOL. We prove that for every minor-closed graph class, model-checking for FOL+DP can be done in quadratic time. We also introduce an extension of FOL+DP, namely the *scattered disjoint paths logic*, FOL+SDP, where we further consider the atomic predicate $s\text{-sdp}_k(x_1, y_1, \dots, x_k, y_k)$, demanding that the disjoint paths are within distance bigger than some fixed value s . Using the same technique we prove that model-checking for FOL+SDP can be done in quadratic time on classes of graphs with bounded Euler genus.

Joint work with Petr A. Golovach and Dimitrios M. Thilikos.

From Twin-Width to Propositional Logic and Back

Stefan Szeider

TU Wien

The width measure twin-width was recently introduced by Bonnet et al. (FOCS 2020), who showed that many NP-hard problems are tractable for graphs of bounded twin-width, generalizing similar results for other width measures, including treewidth and clique-width. In this talk, we discuss two connections between twin-width and propositional logic.

First, we show that a generalization of the propositional satisfiability problem, obtaining the weighted sum of all satisfying assignments to a formula in CNF that set at most k variables to true, is fixed-parameter tractable. As the parameter, we take the certified signed twin-width of the input formula and the bound k .

Second, we propose an efficient encoding of the recognition problem for graphs of bounded twin-width to the propositional satisfiability problem (SAT). This encoding allows us to utilize the power of today's SAT solvers to identify the exact twin-width of several graphs whose twin-width was unknown. The SAT encoding relies on a characterization of twin-width based on elimination sequences. We modify the encoding to deal with the above-mentioned signed twin-width.

Joint work with Robert Ganian, Filip Pokrývka, and André Schidler.

Preprints are available under <https://arxiv.org/abs/2206.01706> (SAT 2022) and <https://arxiv.org/abs/2110.06146> (ALENEX 2022).

From even-hole-free graphs to treewidth

Nicolas Trotignon
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A *hole* in a graph is a chordless cycle of length at least 4. A graph is *even-hole-free* if it does not contain a hole of even length. Even-hole-free graphs attracted some attention because of their analogy with perfect graphs (where holes of odd length together with their complements are excluded). We will survey some results about the structure of even-hole-free graphs, and explain why their structure is so mysterious : on the one hand, the general theorems that are known about them are not strong enough to provide polynomial time algorithms to color or find a maximum stable set; on the other hand, it is quite hard to exhibit example of even hole free that are “complex” in any way.

This remark lead researchers to investigate widths of even-hole-free graphs (treewidth, cliquewidth, rankwidth), with the hope that in some way their width might be restricted. This study failed in the sense that very restricted classes of even-hole graphs turned out to have unbounded width. But it was a success in the sense that it lead to several conjectures about a possible version of the celebrated grid-minor theorem of Robertson and Seymour, with “minor” replaced by “induced subgraph”. It turns out that all these conjectures are now either proved or disproved. We will survey these recent progress.

Killing a Vortex

Sebastian Wiederrecht
IBS DIMAG

The Structural Theorem of the Graph Minors series of Robertson and Seymour asserts that, for every $t \in \mathbb{N}$, there exists some constant c_t such that every K_t -minor-free graph admits a tree decomposition whose torsos can be transformed, by the removal of at most c_t vertices, to graphs that can be seen as the union of some graph that is embeddable to some surface of Euler genus at most c_t and “at most c_t vortices of depth c_t ”. Our main combinatorial result is a “vortex-free” refinement of the above structural theorem as follows: we identify a (parameterized) graph H_t , called *shallow vortex grid*, and we prove that if in the above structural theorem we replace K_t by H_t , then the resulting decomposition becomes “vortex-free”. Up to now, the most general classes of graphs admitting such a result were either bounded Euler genus graphs or the so called single-crossing minor-free graphs. Our result is tight in the sense that, whenever we minor-exclude a graph that is not a minor of some H_t , the appearance of vortices is unavoidable. Using the above decomposition theorem, we design an algorithm that, given an H_t -minor-free graph G , computes the generating function of all perfect matchings of G in polynomial time. This algorithm yields, on H_t -minor-free graphs, polynomial algorithms for computational problems such as the dimer problem, the exact matching problem, and the computation of the permanent. Our results, combined with known complexity results, imply a complete characterization of minor-closed graphs classes where the number of perfect matchings is polynomially computable: They are exactly those graph classes that do not contain every H_t as a minor. This provides a *sharp* complexity dichotomy for the problem of counting perfect matchings in minor-closed classes.

Implicit Representations of Graphs and Randomized Communication

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An *implicit* or *local* representation of a graph $G = (V, E)$ is an assignment $\ell : V \rightarrow \{0, 1\}^*$ of labels to the vertices such that adjacency between any pair $x, y \in V$ can be decided only from the corresponding labels $\ell(x)$ and $\ell(y)$.

A fundamental question is which hereditary classes admit implicit representations with labels of order optimal size. In particular, until recently one of the central questions in the area was the Implicit Graph Conjecture (IGC) positing that any factorial hereditary class (i.e., a class in which the number of n -vertex labeled graphs is $2^{\Theta(n \log n)}$) admits an implicit representation with labels of size $\mathcal{O}(\log n)$.

In this talk we will present a connection between implicit representations of graphs and randomized communication that was developed to make progress towards the IGC and led to the refutation of the conjecture by Hatami and Hatami.

